

<sup>2</sup> Sturms, F. M., Jr., "Nodal period for a circular earth satellite," *ARS J.* **32**, 1037-1039 (1962).

<sup>3</sup> Kalil, F. and Martikan, F., "Derivation of nodal period of an earth satellite and comparisons of several first-order secular oblateness results," *AIAA J.* **1**, 2041-2046 (1963).

<sup>4</sup> Blitzer, L., "The orbit of a satellite in the gravitational field of the earth," *Space Technology Lab. Rept.* 8655-6020-RU000 (1963).

<sup>5</sup> Kozai, Y., "The motion of a close earth satellite," *Astron. J.* **64**, 367-377 (1959).

<sup>6</sup> Merson, R. H., "The motion of a satellite in an axi-symmetric gravitational field," *Geophys. J.* **4**, 17-52 (1961).

<sup>7</sup> Claus, A. J. and Lubowe, A. G., "A high accuracy perturbation method with direct application to communication satellite orbit prediction," *Astronaut. Acta* (to be published).

## Factors Influencing Electrically Sprayed Liquids

J. J. HOGAN,\* R. S. CARSON,† J. M. SCHNEIDER,\* AND  
C. D. HENDRICKS‡

*University of Illinois, Urbana, Ill.*

**E**LECTRICAL spraying was accomplished in vacuo by allowing a liquid to emerge from a metal capillary tube maintained at a high positive potential with respect to a grounded electrode located 1 to 4 cm in front of the capillary tip. Instability and subsequent dispersion of the liquid meniscus into numerous charged particles took place at the tip of the capillary tube. The vacuum surrounding the spraying region was maintained at about  $5 \times 10^{-6}$  torr.

Current distributions of specific charge (charge-to-mass ratio) produced by various liquids were measured under steady-state spraying conditions with a quadrupole mass spectrometer and under naturally pulsed conditions with a time-of-flight mass spectrometer. Only those particles that were sprayed on or near the axis of symmetry of the spectrometer were intercepted and analyzed. Pure liquids and solutions containing suspensions and dissolved ionic conductors were sprayed electrically. The electrical conductivity of the various liquids ranged approximately between  $10^{-4}$  and  $10^{-15}$  mho/cm. No liquid was found which could not be electrically dispersed.

A summary of some of the measurements is presented in Table 1. The specific charge corresponding to the peak of a distribution is called the average specific charge. In general, it was found that the average specific charge produced by the pure liquids was less than for the liquid solutions. Silicone oil, possessing the smallest conductivity of all liquids tested, was the only liquid whose dispersed particles could not be detected. This was probably caused by the emitted particles having specific charges too low to be resolved by the spectrometer, i.e., below 0.001 coul/kg.

High values of specific charge were obtained from a mixture of 2% by weight of Darco grade S-51 activated carbon particles in glycerine and from a mixture of 2% Cab-O-Sil H-5 (a paint thickener composed of silicon dioxide particles) in glycerine. It was also found that under certain conditions the average specific charge produced by dibutyl phthalate

**Table 1 Specific charge measurements for various liquids**

Liquid	Pressure, cm of liq.	Voltage, kv	( <i>q/m</i> ) avg, coul/kg
2% Cab-O-Sil in glycerine	16	17.21	300
2% Carbon in glycerine	30	17	200
Glycerine	17	17	19
	64	4.1	0.42
	58.5	3.8	0.125
2% KOH in glycerine	63.5	3.5	6.74
	1 atm	3.5	3.38
Bu <sub>4</sub> NPi in octoil	60	9	1.67
	120	9	0.71
	10	2.1	0.05
Narcoil-40	62	3.3	0.33
	62	3	0.19
	62	2.1	0.05
Tri cresyl phosphate	65	2.2	0.021
Octoil-S	300	9	0.20
	10	4	0.12
	1 atm	4	0.03
Tetraethylene glycol	60	3	0.20
	1 atm	3	0.097

corresponded to singly charged double molecules (diamers) or about  $10^5$  coul/kg.

The light from a microscope illuminator and an arc lamp was focused at different times onto the emitting region of the capillary tube. This apparently caused sufficient heating of the liquid at the capillary tip to influence the specific-charge distribution. It was found that, as the temperature was increased in steps from 25° to 56°C, the average specific charge for glycerine increased from 0.4 to 3 coul/kg, and the distribution narrowed. This result was not altogether expected. It is possible that photoeffects are present in addition to temperature effects. The temperature was measured with a thermocouple.

In general, the particular spraying mode changed with increasing applied voltage. At voltages near the minimum spraying potential, a single jet of sprayed particles was usually formed along the axis of symmetry. Throughout a higher voltage range, several jets were often observed to spray from the periphery of the capillary tip and to change in number and direction with changes in voltage. Frequently the mode of spraying was such that none of the sprayed particles was intercepted by the spectrometer. Consequently, data could be obtained only for certain applied-voltage ranges. However, it was observed that the average specific charge increased as the applied voltage raised to a power up to about 3.5.

All measurements indicated the average specific charge decreased with an increased hydrostatic liquid pressure or, by Poiseuille's equation, mass-flow rate. At low liquid pressures, the average specific charge tended to remain constant, but the repeatability of data at low pressures was difficult. This phenomenon was attributed to natural pulsations of the spraying mechanism caused by the low flow rate.

The current produced by the spraying process was measured by observing the voltage across a 1-meg resistor inserted in series with the ground electrode. At voltages near the minimum spraying potential, the spraying was generally pulsating in nature and featured the liquid necking down into a slender axial filament from the end of which the spraying actually took place. It is possible that a particular combination of capillary size, voltage, pressure, spacing, and other parameters will allow continuous and nonpulsating spraying. Although such a combination so far cannot be predicted theoretically, it can be obtained experimentally.

The current pulsing that occurred when glycerine was sprayed was strongly dependent upon the accelerating voltage.

Presented as Preprint 64-12 at the AIAA Aerospace Sciences Meeting, New York, January 20-20, 1964; revision received April 8, 1964. This work supported in part by Air Force Office of Scientific Research Grant 107-63 and in part by National Science Foundation Grant 19776.

\* Instructor, Department of Electrical Engineering, Charged Particle Research Laboratory.

† Research Associate, Department of Electrical Engineering, Charged Particle Research Laboratory.

‡ Professor, Department of Electrical Engineering, Charged Particle Research Laboratory. Member AIAA.

Measurements indicated that the period, width, and amplitude of the current pulses decreased as the voltage was increased, with minor fluctuations occurring in their values. Changing the liquid pressure had a relatively small effect on the amplitude and width of the current pulses, but it had a marked effect on the pulsing period. It was also observed that the minimum spraying potential decreased as the liquid pressure was increased.

As the voltage was increased, the period-to-width ratio decreased to a value of about three. At this point, the d.c. spray-current level suddenly jumped to a new value, indicating an essentially constant current. The current pulses themselves disappeared. As the voltage was further increased, the new d.c. level also increased, and the current pulses reappeared. More than one set of current pulses was now present, not necessarily of the same amplitude and frequency, and sprays were located at different points on the periphery of the capillary tip. Increasing the voltage caused additional current pulses, smaller in amplitude and width, until the current looked like a jumble of noise.

Using a larger diameter capillary or increased spacing between capillary and grounded electrode gave results similar to the foregoing but translated to higher voltage values.

The naturally pulsed spraying, which was characteristic of low voltages and low liquid pressures, made the use of a time-of-flight mass spectrometer attractive for determining specific-charge spectra. A positive voltage was applied to an electrode at the entrance of the instrument so that sprayed droplets entered the drift space only when a negative control pulse was applied to the electrode. The control pulse was synchronized with the current pulse produced by the spraying glycerine, and its relative position could be varied at will. The drift time, and hence the specific-charge values, were accurately measured by using a narrow control pulse.

A series of measurements on the pulsed spraying of glycerine at low voltages indicated that the specific charges produced at the peaks of the current pulses were lower and more widely spread in values than at the beginning and end of the current pulses. A typical specific-charge range was 0.2 to 1.8 coul/kg with a few random values up to about 10 coul/kg.

The various forementioned measurements show that the disruptive effects of electric fields are important in the field emission of macroscopic liquid droplets. Not only the initial instability of the liquid surface, which generates the charged particles, but also the stability of a preformed charged particle injected into an accelerating field is of interest. For a rigid sphere of radius  $a$  in a uniform electric field  $E$ , it can be shown that equating surface-tension pressure to maximum electric stress on the sphere (the point of instability) results in

$$\frac{9}{4} aE^2 + \frac{a^2\rho}{2\epsilon_0} \left(\frac{q}{m}\right) + \frac{a^3\rho^2}{36\epsilon_0^2} \left(\frac{q}{m}\right)^2 - \frac{\gamma}{\epsilon_0} = 0 \quad (1)$$

where  $\rho$  is the mass density,  $m$  the mass,  $\gamma$  the coefficient of surface tension, and  $q$  the total charge on the sphere. The following two cases are immediately of interest.

Case 1:  $q = 0$ . In this case, Eq. (1) simplifies to

$$\frac{9}{4} aE^2 = \gamma/\epsilon_0$$

or

$$E(a)^{1/2} = \frac{2}{3}(\gamma/\epsilon_0)$$

This becomes, for water with  $\gamma = 58$  dynes/cm,  $E(a)^{1/2} = 5400$  v/cm<sup>1/2</sup>. Experimental measurements on the bursting of soap bubbles show that  $E(a)^{1/2} = 3600$  v/cm<sup>1/2</sup>. The difference here may be attributed to the fact that the soap bubbles depart from the spherical shape before becoming unstable. This occurs because the prolate spheroidal shape is one of less stored energy than the spherical shape in this case.

Case 2: In order for the quadratic equation (1) to have positive solutions for  $E$  (the only meaningful solutions), it is found that

$$\gamma > \frac{a^3\rho^2}{36\epsilon_0} \left(\frac{q}{m}\right)^2$$

which is the well-known Rayleigh limit for charged droplets.

As a final example of the instability of a charged particle in a uniform electric field, suppose  $a = 10^{-8}$  m,  $(q/m) = 10^3$  coul/kg,  $\rho = 10^3$  kg/m<sup>3</sup>, and  $\gamma = 0.1$  newtons/m. These values are typical for the organic liquids in present use. Then  $E = 5.4 \times 10^8$  v/m, which value is high. Hence, in order to use an accelerating potential of  $10^6$  v, the spacing should probably be greater than 0.3 cm. For a higher specific charge with the same droplet, this distance would have to be greater.

## Gaseous Nuclear Rocket with MHD Vortex Fuel Containment

K. O. KESSEY\* AND R. A. GROSS†  
Columbia University, New York, N. Y.

### I. Introduction

THE potentialities of a nuclear rocket are well known,<sup>1-3</sup> but many difficult and fundamental problems remain to be solved before an attempt is made to build a practical device. One of the major problems is the separation of the gaseous fuel (for example, uranium 235) from the propellant (hydrogen). It was this particular aspect of the problem which motivated the previous study by Gross and Kessey.<sup>5</sup> In that study, a hydromagnetically driven vortex separation system is analyzed with primary attention given to the magnetohydrodynamics of a cylindrical system.

In the present note, we examine some potentialities and limitations of this concept for a practical nuclear rocket system. We determine the size, flow limitations, composition of the exhaust gas, electrical power, reactor power, etc. required by a hydromagnetic vortex, gaseous nuclear rocket reactor. The pertinent analytical results of Refs. 4 and 5 are applied to a practical system. The simultaneous requirements for nuclear criticality, adequate fuel-propellant separation, input composition to maintain a steady state, and limits on electrical powerplant weight form the crux of this analysis of a magnetohydrodynamic vortex nuclear rocket.

### II. Physical Conditions for Steady Nuclear Critical Flow

The relevant hydromagnetic equations that we have employed are lengthy and can be found in Refs. 4-6. The notation used here is identical to that described in these papers. There are several simultaneous physical constraints in the proposed system which must be satisfied when the relevant equations [for example, (5, 32, 38, 47, and 48) of Ref. 5, or (1, 8, 10, and 11) of Ref. 6] are evaluated for the gaseous nuclear rocket. These constraints are briefly:

1) The radial mass flow rate (i.e.,  $G_0$  or radial Reynolds number  $R_N$ ) must be such that the solutions of the equations yield positive values for the relative composition. There is a limit on the radial Reynolds number for which  $n_U/n_{O_2}$  is positive. The same type of limit on radial mass flow was

Received February 17, 1964. This study was supported by the Air Force Office of Scientific Research under Grant AF AFOSR-48-63 and Contract AF 49(638)-1254. This note is an extension of Ref. 5. It is a condensed version of Ref. 6, a Columbia University Plasma Laboratory Report.

\* Graduate Student and Research Investigator.

† Professor of Engineering Science. Fellow Member AIAA